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FLUID MECHANICS

in SI Units



 Pearson

R. C. HIBBELER

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FLUID MECHANICS

in SI Units

R. C. Hibbeler

SI Conversion by
Kai Beng Yap



To the Student

With the hope that this work will stimulate
an interest in Fluid Mechanics
and provide an acceptable guide to its understanding.

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PREFACE

This book has been written and revised several times over a period of nine years, in order to further improve its contents and account for the many suggestions and comments from my students, university colleagues, and reviewers. It is hoped that this effort will provide those who use this work with a clear and thorough presentation of both the theory and application of fluid mechanics. To achieve this objective, I have incorporated many of the pedagogic features that I have used in my other books. These include the following:

Organization and Approach. Each chapter is organized into well-defined sections that contain an explanation of specific topics, illustrative example problems, and at the end of the chapter, a set of relevant homework problems. The topics within each section are placed into subgroups defined by boldface titles. The purpose of this organization is to present a structured method for introducing each new definition or concept, and to make the book a convenient resource for later reference and review.

Procedures for Analysis. This unique feature provides the student with a logical and orderly method to follow when applying the theory that has been discussed in a particular section. The example problems are then solved using this outlined method in order to clarify its numerical application. Realize, however, that once the relevant principles have been mastered, and enough confidence and judgment has been obtained, the student can then develop his or her own procedures for solving problems.

Important Points. This feature provides a review or summary of the most important concepts in a section, and highlights the most significant points that should be remembered when applying the theory to solve problems. A further review of the material is given at the end of the chapter.

Photos. The relevance of knowing the subject matter is reflected by the realistic applications depicted in the many photos placed throughout the book. These photos are often used to show how the principles of fluid mechanics apply to real-world situations.

Fundamental Problems. These problem sets are selectively located just after the example problems. They offer students simple applications of the concepts and therefore provide them with the chance to develop their problem-solving skills before attempting to solve any of the standard problems that follow. Students may consider these problems as extended examples, since they all have complete solutions and answers given in the back of the book. Additionally, the fundamental problems offer students an excellent means of preparing for exams, and they can be used at a later time to prepare for the Fundamentals in Engineering Exam.

Homework Problems. The majority of problems in the book depict realistic situations encountered in engineering practice. It is hoped that this realism will both stimulate interest in the subject, and provide a means for developing the skills to reduce any problem from its physical description to a model or symbolic representation to which the principles of fluid mechanics may then be applied.

An attempt has been made to arrange the problems in order of increasing difficulty. Except for every fourth problem, indicated by an asterisk (*), the answers to all the other problems are given in the back of the book.

Accuracy. Apart from my work, the accuracy of the text and problem solutions have all been thoroughly checked by other parties. Most importantly, Kai Beng Yap, Kurt Norlin along with Bittner Development Group, as well as James Liburdy, Jason Wexler, Maha Haji, and Brad Saund.

Contents

The book is divided into 14 chapters. Chapter 1 begins with an introduction to fluid mechanics, followed by a discussion of units and some important fluid properties. The concepts of fluid statics, including constant accelerated translation of a liquid and its constant rotation, are covered in Chapter 2. In Chapter 3, the basic principles of fluid kinematics are covered. This is followed by the continuity equation in Chapter 4, the Bernoulli and energy equations in Chapter 5, and fluid momentum in Chapter 6. In Chapter 7, differential fluid flow of an ideal fluid is discussed. Chapter 8 covers dimensional analysis and similitude. Then the viscous flow between parallel plates and within pipes is treated in Chapter 9. The analysis is extended to Chapter 10 where the design of pipe systems is discussed. Boundary layer theory, including topics related to pressure drag and lift, is covered in Chapter 11. Chapter 12 discusses open channel flow, and Chapter 13 covers a variety of topics in compressible flow. Finally, turbomachines, such as axial and radial flow pumps and turbines are treated in Chapter 14.

Alternative Coverage. After covering the basic principles of Chapters 1 through 6, at the discretion of the instructor, the remaining chapters may be presented in *any sequence*, without the loss of continuity. If time permits, sections involving more advanced topics, may be included in the course. Most of these topics are placed in the later chapters of the book. In addition, this material also provides a suitable reference for basic principles when it is discussed in more advanced courses.

Acknowledgments

I have endeavored to write this book so that it will appeal to both the student and instructor. Through the years many people have helped in its development, and I will always be grateful for their valued suggestions and comments. During the past years, I have had the privilege to teach my students during the summer at several German universities, and in particular I would like to thank Prof. H. Zimmermann at the University of Hanover, Prof. F. Zunic of the Technical University in Munich, and Prof. M. Raffel at the Institute of Fluid Mechanics in Goettingen, for their assistance. In addition, I. Vogelsang and Prof. M. Geyh of the University of Mecklenburg have provided me with logistic support in these endeavors. I would also like to thank Prof. K. Cassel at Illinois Institute of Technology, Prof. A. Yarin at the University of Illinois-Chicago, and Dr. J. Gotlieb for their comments and suggestions. In addition, the following individuals have contributed important reviewer comments relative to preparing this work:

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There are a few people that I feel deserve particular recognition. A longtime friend and associate, Kai Beng Yap, was of great help in checking the entire manuscript, and helping to further check all the problems. And a special note of thanks also goes to Kurt Norlin for his diligence and support in this regard. During the production process I am also thankful for the support of my long time Production Editor, Rose Kernan, and my Managing Editor, Scott Disanno. My wife, Conny, and

daughter, Mary Ann, have been a big help with the proofreading and typing needed to prepare the manuscript for publication.

Lastly, many thanks are extended to all my students who have given me their suggestions and comments. Since this list is too long to mention, it is hoped that those who have helped in this manner will accept this anonymous recognition.

I value your judgment as well, and would greatly appreciate hearing from you if at any time you have any comments or suggestions that may help to improve the contents of this book.

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Indian Adaptation

The publishers would like to thank Dr Pravin Kumar, Department of Mechanical Engineering, Delhi Technological University, Delhi for his valuable suggestions and inputs for enhancing the content of this book to suit the requirement of Indian universities.

Resources for Instructors

- **Instructor's Solutions Manual.** An instructor's solutions manual was prepared by the author. The manual includes homework assignment lists and was also checked as part of the accuracy checking program. The Instructor Solutions Manual is available at www.pearsoned.co.in/rchibbeler.
- **Presentation Resource.** All art from the text is available in PowerPoint slide and JPEG format. These files are available for download from the Instructor Resource Center at www.pearsoned.co.in/rchibbeler.

ABOUT THE AUTHOR

R.C. Hibbeler graduated from the University of Illinois at Urbana with a BS in Civil Engineering (majoring in Structures) and an MS in Nuclear Engineering. He obtained his PhD in Theoretical and Applied Mechanics from Northwestern University. Professor Hibbeler's professional experience includes postdoctoral work in reactor safety and analysis at Argonne National Laboratory, and structural and stress analysis work at Chicago Bridge and Iron, as well as at Sargent and Lundy in Chicago. He has practiced engineering in Ohio, New York, and Louisiana.

Professor Hibbeler currently teaches both civil and mechanical engineering courses at the University of Louisiana– Lafayette. In the past, he has taught at the University of Illinois at Urbana, Youngstown State University, Illinois Institute of Technology, and Union College.

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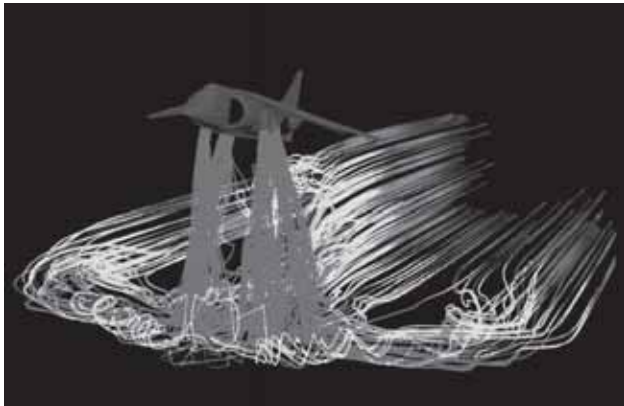
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FLUID MECHANICS

in SI Units

Chapter 1



(©AZybr/Shutterstock)

Fluid mechanics plays an important role in the design and analysis of pressure vessels, pipe systems, and pumps used in chemical processing plants.

Fundamental Concepts

CHAPTER OBJECTIVES

- To provide a description of fluid mechanics and indicate its various branches.
- To explain how matter is classified as a solid, liquid, or gas.
- To discuss the system of units for measuring fluid quantities, and establish proper calculation techniques.
- To define some important fluid properties, such as density, specific weight, bulk modulus, and viscosity.
- To describe the concepts of vapor pressure, surface tension, and capillarity.

1.1 Introduction

Fluid mechanics is a study of the behavior of fluids that are either at rest or in motion. It is one of the primary engineering sciences that has important applications in many engineering disciplines. For example, aeronautical and aerospace engineers use fluid mechanics principles to study flight, and to design propulsion systems. Civil engineers use this subject to design drainage channels, water networks, sewer systems, and water-resisting structures such as dams and levees. Fluid mechanics is used by mechanical engineers to design pumps, compressors, turbines, process control systems, heating and air conditioning equipment, and to design wind turbines and solar heating devices. Chemical and petroleum engineers apply this subject to design equipment used for filtering, pumping, and mixing fluids. And finally, engineers in the electronics and computer industry use fluid mechanics principles to design switches, screen displays, and data storage equipment. Apart from the engineering profession, the principles of fluid mechanics are also used in biomechanics, where it plays a vital role in the understanding of the circulatory, digestive, and respiratory systems, and in meteorology to study the motion and effects of tornadoes and hurricanes.

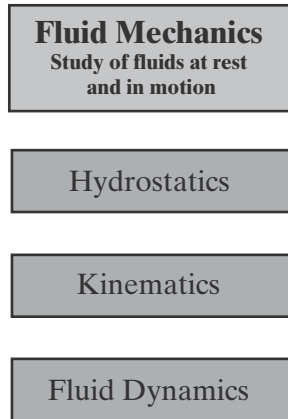


Fig. 1-1

Branches of Fluid Mechanics. The principles of fluid mechanics are based on Newton's laws of motion, the conservation of mass, the first and second laws of thermodynamics, and laws related to the physical properties of a fluid. The subject is divided into three main categories, as shown in Fig. 1-1.

- **Hydrostatics** considers the forces acting on a fluid at rest.
- **Fluid kinematics** is the study of the geometry of fluid motion.
- **Fluid dynamics** considers the forces that cause acceleration of a fluid.

Historical Development. A fundamental knowledge of the principles of fluid mechanics has been of considerable importance throughout the development of human civilization. Historical records show that through the process of trial and error, early societies, such as the Roman Empire, used fluid mechanics in the construction of their irrigation and water supply systems. In the middle of the 3rd century B.C., Archimedes discovered the principle of buoyancy, and then much later, in the 15th century, Leonardo Da Vinci developed principles for the design of canal locks and other devices used for water transport. However, the greatest discoveries of basic fluid mechanics principles occurred during the 16th and 17th centuries. It was during this period that Evangelista Torricelli designed the barometer, Blaise Pascal formulated the law of static pressure, and Isaac Newton developed his law of viscosity to describe the nature of fluid resistance to flow.

In the 1700s, Leonhard Euler and Daniel Bernoulli pioneered the field of **hydrodynamics**, a branch of mathematics dealing with the motion of an idealized fluid, that is, one having a constant density and providing no internal frictional resistance. Unfortunately, hydrodynamic principles could not be used by engineers to study some types of fluid motion, since the physical properties of the fluid were not fully taken into account. The need for a more realistic approach led to the development of **hydraulics**. This field uses empirical equations found from fitting curves to data determined from experiments, primarily for applications involving water. Contributors included Gustave Coriolis, who developed water turbines, and Gotthilf Hagen and Jean Poiseuille, who studied the resistance to water flowing through pipes. In the early 20th century, hydrodynamics and hydraulics were essentially *combined* through the work of Ludwig Prandtl, who introduced the concept of the boundary layer while studying aerodynamics. Through the years, many others have also made important contributions to this subject, and we will discuss many of these throughout the text.*

*References [1] and [2] provide a more complete description of the historical development of this subject.

1.2 Characteristics of Matter

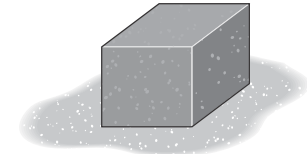
In general, matter can be classified by the state it is in—as a solid, a liquid, or a gas.

Solid. A *solid* maintains a definite shape and volume, Fig. 1–2a. It maintains its shape because the molecules or atoms of a solid are densely packed and are held tightly together, generally in the form of a lattice or geometric structure. The spacing of atoms within this structure is due in part to large cohesive forces that exist between molecules. These forces prevent any relative movement, except for any slight vibration of the molecules themselves. As a result, when a solid is subjected to a load it will not easily deform, but once in its deformed state, it will continue to support the load.

Liquid. A *liquid* is composed of molecules that are more spread out than those in a solid. Their intermolecular forces are weaker, so liquids do not hold their shape. Instead, they *flow* and take the shape of their container, Fig. 1–2b. Although liquids can easily deform, their molecular spacing allows them to resist compressive forces when they are confined.

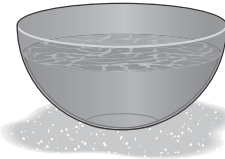
Gas. A *gas* is a substance that fills the entire volume of its container, Fig. 1–2c. Gases are composed of molecules that are much farther apart than those of a liquid. As a result, the molecules of a gas are free to travel away from one another until a force of repulsion pushes them away from other gas molecules or from the molecules on the surface of a solid or liquid boundary.

Definition of a Fluid. Liquids and gases are classified as *fluids* because they are substances that continuously deform or flow when subjected to a shear or tangential force. This behavior is shown on small fluid elements in Fig. 1–3, where a plate moves over the top surface of the fluid. The deformation of the fluid will continue as long as the shear force is applied, and once it is removed, the fluid will keep its new shape rather than returning to its original one. In this text we will only concentrate on those substances that exhibit *fluid behavior*, meaning any substance that will flow because it cannot support a shear loading, *regardless* of how *small* the shear force is, or how *slowly* the “fluid” deforms.



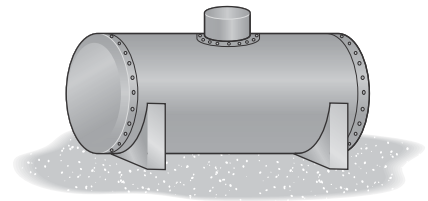
Solids maintain a constant shape

(a)



Liquids take the shape of their container

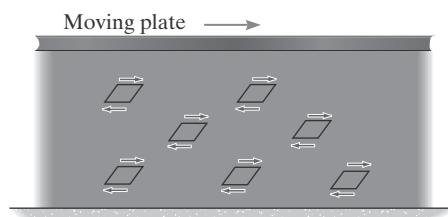
(b)



Gases fill the entire volume of their container

(c)

Fig. 1–2



All fluid elements *deform* when subjected to shear

Fig. 1–3

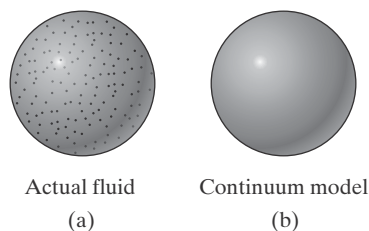


Fig. 1-4

Continuum. Studying the behavior of a fluid by analyzing the motion of all its many molecules would be an impossible task, Fig. 1-4a. Fortunately, however, almost all engineering applications involve a volume of fluid that is much greater than the distance between adjacent molecules of the fluid, and so it is reasonable to assume the fluid is uniformly dispersed and continuous throughout this volume. Under these circumstances, we can then consider the fluid to be a **continuum**, that is, a continuous distribution of matter leaving no empty space, Fig. 1-4b. This assumption allows us to use *average properties* of the fluid at any point within the volume the fluid occupies. For those situations where the molecular distance does become important, which is on the order of a billionth of a meter, the continuum model does not apply, and it is necessary to employ statistical techniques to study the fluid flow, a topic that will not be considered here. See Ref. [3].

1.3 The International System of Units

There are five basic quantities primarily used in fluid mechanics: length, time, mass, force, and temperature. Of these, length, time, mass, and force are all *related* by Newton's second law of motion, $F = ma$. As a result, the *units* used to define the size of these quantities cannot *all* be selected arbitrarily. The equality $F = ma$ is maintained when *three* of these units are arbitrarily defined, and the fourth unit is then *derived* from the equation.

The International System of units, abbreviated as SI after the French term *Système International d'Unités*, is a modern version of the metric system that has received worldwide recognition. As shown in Table 1-1, the SI system specifies length in meters (m), time in seconds (s), and mass in kilograms (kg). The unit of force, called a newton (N), is *derived* from $F = ma$, where 1 newton is equal to the force required to give 1 kilogram of mass an acceleration of 1 m/s^2 ($\text{N} = \text{kg} \cdot \text{m/s}^2$), Fig. 1-5a.

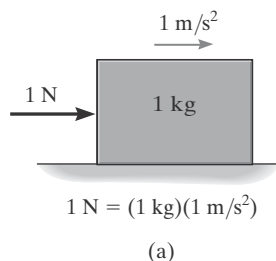


Fig. 1-5

TABLE 1-1 International System of Units

Quantity	Length	Time	Mass	Force	Temperature	
SI Units	meter m	second s	kilogram kg	Newton* N $\left(\frac{\text{kg} \cdot \text{m}}{\text{s}^2}\right)$	Kelvin K	Celsius °C
*Derived unit.						

Weight. To determine the weight of a fluid in newtons at the “standard location,” where the acceleration due to gravity is $g = 9.81 \text{ m/s}^2$, and the mass of the fluid is m (kg), we have

$$W \text{ (N)} = [m \text{ (kg)}] (9.81 \text{ m/s}^2) \quad (1-1)$$

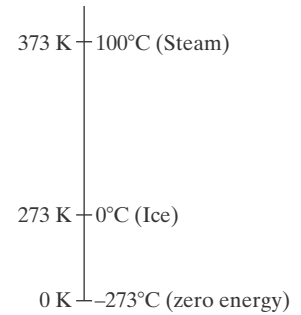
And so a fluid having a mass of 1 kg has a weight of 9.81 N, 2 kg of fluid has a weight of 19.62 N, and so on.

Temperature. The absolute temperature is the temperature measured from a point where the molecules of a substance have so called “zero energy”.* The unit for *absolute temperature* in the SI system is the kelvin (K). This unit is expressed *without* reference to degrees, so 7 K is stated as “seven kelvins.” Although not officially an SI unit, an equivalent size unit measured in degrees Celsius ($^{\circ}\text{C}$) is often used. This measurement is referenced from the freezing and boiling points of water, where the freezing point is at 0°C (273 K) and the boiling point is at 100°C (373 K), Fig. 1–5*b*. For conversion,

$$T_K = T_C + 273 \quad (1-2)$$

Equations 1–1 and 1–2 will be used in this text since they are suitable for most engineering applications. However, use the exact value of 273.15 K in Eq. 1–2 for more accurate work. Also, at the “standard location,” the more exact value $g = 9.807 \text{ m/s}^2$ or the *local* acceleration due to gravity should be used in Eq. 1–1.

Prefixes. When a numerical quantity is either very large or very small, the units used to define its size should be modified by using a prefix. The range of prefixes used for problems in this text is shown in Table 1–2. Each



The Kelvin and Celsius scales
(b)

Fig. 1–5 (cont.)

TABLE 1–2 Prefixes

	Exponential Form	Prefix	SI Symbol
<i>Submultiple</i>			
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	μ
0.000 000 001	10^{-9}	nano	n
<i>Multiple</i>			
1 000 000 000	10^9	Giga	G
1 000 000	10^6	Mega	M
1 000	10^3	kilo	k

*This is actually an unreachable point according to the law of quantum mechanics.

represents a multiple or submultiple of a unit that moves the decimal point of a numerical quantity either forward or backward by three, six, or nine places. For example, $5\,000\,000\text{ g} = 5000\text{ kg}$ (kilogram) = 5 Mg (Megagram), and $0.000\,006\text{ s} = 0.006\text{ ms}$ (millisecond) = $6\text{ }\mu\text{s}$ (microsecond).

As a general rule, quantities defined by several units that are multiples of one another are separated by a dot to avoid confusion with prefix notation. Thus, $\text{m} \cdot \text{s}$ is a meter-second, whereas ms is a millisecond. And finally, the exponential power applied to a unit having a prefix refers to *both the unit and its prefix*. For example, $\text{ms}^2 = (\text{ms})^2 = (\text{ms})(\text{ms}) = (10^{-3}\text{s})(10^{-3}\text{s}) = 10^{-6}\text{s}^2$.

1.4 Calculations

Application of fluid mechanics principles often requires algebraic manipulations of a formula followed by numerical calculations. For this reason it is important to keep the following concepts in mind.

Dimensional Homogeneity. The terms of an equation used to describe a physical process must be *dimensionally homogeneous*, that is, each term must be expressed in the *same units*. Provided this is the case, then all the terms of the equation can be *combined* when numerical values are substituted for the variables. For example, consider the Bernoulli equation, which is a specialized application of the principle of work and energy. We will study this equation in Chapter 5, but it can be expressed as

$$\frac{p}{\gamma} + \frac{V^2}{2g} + z = \text{constant}$$

Here, the pressure p is expressed in N/m^2 , the specific weight γ is in N/m^3 , the velocity V is in m/s , the acceleration due to gravity g is in m/s^2 , and the elevation z is in meters, m . *Regardless of how this equation is algebraically arranged, it must maintain its dimensional homogeneity.* In the form stated, each of the three terms is in meters, as noted by a cancellation of units in each fraction.

$$\frac{\text{N}/\text{m}^2}{\text{N}/\text{m}^3} + \frac{(\text{m}/\text{s})^2}{\text{m}/\text{s}^2} + \text{m}$$

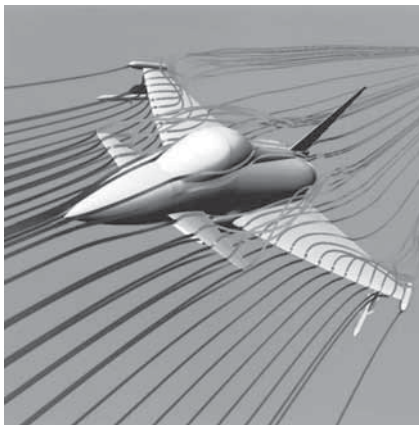
Because almost all problems in fluid mechanics involve the solution of dimensionally homogeneous equations, a *partial check* of the algebraic

manipulation of any equation can therefore be made by checking to see if all the terms have the *same units*.

Calculation Procedure. When performing numerical calculations, *first* represent all the quantities in terms of their base or derived units by converting any prefixes to powers of 10. Then do the calculation, and finally express the result using a *single prefix*. For example, $3 \text{ MN}(2 \text{ mm}) = [3(10^6) \text{ N}][2(10^{-3}) \text{ m}] = 6(10^3) \text{ N} \cdot \text{m} = 6 \text{ kN} \cdot \text{m}$.

In the case of fractional units, with the exception of the kilogram, the prefix should always be in the numerator, as in MN/s or mm/kg. Also, after the calculation, it is best to keep numerical values between 0.1 and 1000; otherwise, a suitable prefix should be chosen.

Accuracy. Numerical work in fluid mechanics is almost always performed using pocket calculators and computers. It is important, however, that the answers to any problem be reported with justifiable accuracy using an appropriate number of significant figures. As a general rule, always retain more digits in your calculations than are given in the problem data. Then round off your final answer to *three significant figures*, since data for fluid properties and many experimental measurements are often reported with this accuracy. We will follow this procedure in this text, where the intermediate calculations for the example problems will often be worked out to four or five significant figures, and then the answers will generally be reported to *three significant figures*.



Complex flows are often studied using a computer analysis; however, it is important to have a good grasp of the principles of fluid mechanics to be sure reasonable predictions have been made. (© CHRIS SATTLBERGER/Science Source)

1.5 Problem Solving

At first glance, the study of fluid mechanics can be rather daunting, because there are many aspects of this field that must be understood. Success at solving problems, however, will depend on your attitude and your willingness to both focus on class lectures and to carefully read the material in the text. Aristotle once said, “What we have to learn to do, we learn by doing,” and indeed *your ability to solve problems* in fluid mechanics depends upon a thoughtful preparation and neat presentation.

In any engineering subject, it is very important that you follow a logical and orderly procedure when solving problems. In the case of fluid mechanics this should include the sequence of steps outlined below:

General Procedure for Analysis

Fluid Description.

Fluids can behave in many different ways, and so at the outset it is important to *identify the type of fluid flow* and specify the fluid’s *physical properties*. Knowing this provides a means for the proper selection of equations used for an analysis.

Analysis.

This generally involves the following steps:

- Tabulate the problem data and draw, to a reasonably large scale, any necessary diagrams.
- Apply the relevant principles, generally in mathematical form. When substituting numerical data into any equations, be sure to include their units, and check to be sure the terms are dimensionally homogeneous.
- Solve the equations, and report any numerical answers to three significant figures.
- Study the answer with technical judgment and common sense to determine whether or not it seems reasonable.

When applying this procedure, do the work as neatly as possible. Being neat generally stimulates clear and orderly thinking, and vice versa.

Important Points

- Solids have a definite shape and volume, liquids take the shape of their container, and gases fill the entire volume of their container.
- Liquids and gases are fluids because they continuously deform or flow when subjected to a shear force, no matter how small this force is.
- For most engineering applications, we can consider a fluid to be a continuum, and therefore use its average properties to model its behavior.
- Weight is measured in newtons and is determined from $W \text{ (N)} = [m \text{ (kg)}] (9.81 \text{ m/s}^2)$.
- Certain rules must be followed when performing calculations and using prefixes. First convert all numerical quantities with prefixes to their base units, then perform the calculations, and finally choose an appropriate prefix for the result.
- The derived equations of fluid mechanics are all dimensionally homogeneous, and thus each term in an equation has the same units. Careful attention should therefore be paid to the units when entering data and then solving an equation.
- As a general rule, perform calculations with sufficient numerical accuracy, and then round off the final answer to three significant figures.

EXAMPLE 1.1

Evaluate $(80 \text{ MN/s})(5 \text{ mm})^2$, and express the result with SI units having an appropriate prefix.

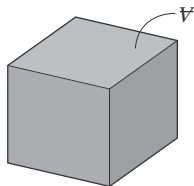
SOLUTION

We first convert all the quantities with prefixes to powers of 10, perform the calculation, and then choose an appropriate prefix for the result.

$$\begin{aligned}
 (80 \text{ MN/s})(5 \text{ mm})^2 &= [80(10^6) \text{ N/s}][5(10^{-3}) \text{ m}]^2 \\
 &= [80(10^6) \text{ N/s}][25(10^{-6}) \text{ m}^2] \\
 &= 2(10^3) \text{ N} \cdot \text{m}^2/\text{s} = 2 \text{ kN} \cdot \text{m}^2/\text{s} \quad \text{Ans.}
 \end{aligned}$$

1.6 Basic Fluid Properties

Assuming the fluid to be a continuum, we will now define some important physical properties that are used to describe it.



Density is
mass/volume

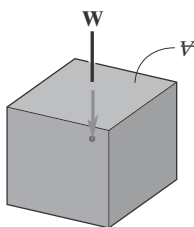
Fig. 1-6

Density. The *density* ρ (rho) refers to the mass of the fluid that is contained in a unit of volume, Fig. 1-6. It is measured in kg/m^3 and is determined from

$$\rho = \frac{m}{V} \quad (1-3)$$

Here m is the mass of the fluid, and V is its volume.

Liquid. Through experiment it has been found that a liquid is practically incompressible, that is, the density of a liquid varies little with pressure. It does, however, have a slight but greater variation with temperature. For example, water at 4°C has a density of $\rho_w = 1000 \text{ kg}/\text{m}^3$, whereas at 100°C , $\rho_w = 958.1 \text{ kg}/\text{m}^3$. For most practical applications, provided the temperature range is small, we can therefore consider *the density of a liquid to be essentially constant*.



Specific weight is
weight/volume

Fig. 1-7

Gas. Unlike a liquid, temperature and pressure can markedly affect the density of a gas, since it has a higher degree of compressibility. For example, air has a density of $\rho = 1.23 \text{ kg}/\text{m}^3$ when the temperature is 15°C and the atmospheric pressure is 101.3 kPa [1 Pa (pascal) = $1 \text{ N}/\text{m}^2$]. But at this same temperature, and at twice the pressure, the density of air *doubles* and becomes $\rho = 2.46 \text{ kg}/\text{m}^3$.

Appendix A lists typical values for the densities of common liquids and gases. Included are tables of specific values for water at different temperatures, and air at different temperatures and elevations.

Specific Weight. The *specific weight* γ (gamma) of a fluid is its weight per unit volume, Fig. 1-7. It is measured in N/m^3 . Thus,

$$\gamma = \frac{W}{V} \quad (1-4)$$

Here, W is the weight of the fluid, and V is its volume.

Since weight is related to mass by $W = mg$, then substituting this into Eq. 1-4, and comparing this result with Eq. 1-3, the specific weight is related to the density by

$$\gamma = \rho g \quad (1-5)$$

Typical values of specific weights for common liquids and gases are also listed in Appendix A.

Specific Gravity. The *specific gravity* S of a substance is a dimensionless quantity that is defined as the ratio of its density or specific weight to that of some other substance that is taken as a “standard.” It is most often used for liquids, and water at an atmospheric pressure of 101.3 kPa and a temperature of 4°C is taken as the standard. Thus,

$$S = \frac{\rho}{\rho_w} = \frac{\gamma}{\gamma_w} \quad (1-6)$$

The density of water for this case is $\rho_w = 1000 \text{ kg/m}^3$, and its specific weight is 9.81 kN/m^3 . So, for example, if an oil has a density of $\rho_o = 880 \text{ kg/m}^3$, then its specific gravity will be $S_o = 0.880$.

Ideal Gas Law. In this text we will consider every gas to behave as an *ideal gas*.* Such a gas is assumed to have enough separation between its molecules so that the molecules have no attraction to one another. Also, the gas must not be near the point of condensation into either a liquid or a solid state.

From experiments, mostly performed with air, it has been shown that ideal gases behave according to the *ideal gas law*. It can be expressed as

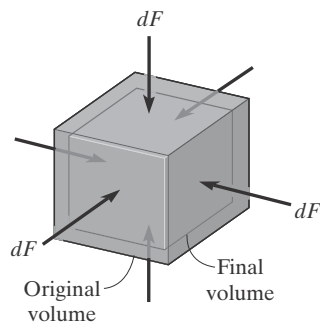
$$p = \rho RT \quad (1-7)$$

Here, p is the *absolute pressure*, or force per unit area, referenced from a perfect vacuum, ρ is the density of the gas, R is the gas constant, and T is the *absolute temperature*. Typical values of R for various gases are given in Appendix A. For example, for air, $R = 286.9 \text{ J/(kg} \cdot \text{K)}$, where $1 \text{ J (joule)} = 1 \text{ N} \cdot \text{m}$.

*Nonideal gases and vapors are studied in thermodynamics.



The volume, pressure, and temperature of the gas in this tank are related by the ideal gas law.



Bulk modulus

Fig. 1-8

Bulk Modulus. The *bulk modulus of elasticity*, or simply the **bulk modulus**, is a measure of the amount by which a fluid offers a resistance to compression. To define this property, consider the cube of fluid in Fig. 1-8, where each face has an area A and is subjected to an incremental force dF . The intensity of this force per unit area is the *pressure*, $dp = dF/A$. As a result of this pressure, the original volume V of the cube will *decrease* by dV . This incremental pressure, divided by this decrease in volume per unit volume, dV/V , defines the bulk modulus, namely,

$$E_V = -\frac{dp}{dV/V} \quad (1-8)$$

The minus sign is included to show that the *increase* in pressure (positive) causes a decrease in volume (negative).

The units for E_V are the same as for pressure—that is, force per area—since the volume ratio is dimensionless. Typical unit is N/m^2 or Pa.

Liquid. Because the density of a liquid changes very little with pressure, its bulk modulus is very high. For example, sea water at atmospheric pressure and room temperature has a bulk modulus of about $E_V = 2.20 \text{ GPa}$.* If we use this value and consider the deepest region of the Pacific Ocean, where the water pressure is 110 MPa, then Eq. 1-8 shows that the fractional compression of water is only $\Delta V/V = [110(10^6) \text{ Pa}] / [2.20(10^9) \text{ Pa}] = 5.0\%$. For this reason, we can assume that for most practical applications, *liquids can be considered incompressible*, and, as stated previously, their density remains constant.**

*Of course, solids can have much higher bulk moduli. For example, the bulk modulus for steel is 160 GPa.

**The *compressibility* of a flowing liquid must, however, be considered for some types of fluid analysis. For example, “water hammer” is created when a valve on a pipe is suddenly closed. This causes an abrupt local change in density of the water near the valve, which generates a pressure wave that travels down the pipe and produces a hammering sound when the wave encounters a bend or other obstruction in the pipe. See Ref. [7].

Gas. A gas, because of its low density, is thousands of times more compressible than a liquid, and so its bulk modulus will be much smaller. For a gas, however, the relation between the applied pressure and the volume change depends upon the process used to compress the gas. Later, in Chapter 13, we will study this effect as it relates to compressible flow, where changes in pressure become significant. However, if the gas flows at *low velocities*, that is, less than about 30% the speed of sound in the gas, then only *small changes* in the gas pressure occur, and so, even with its low bulk modulus, at constant temperature a gas, like a liquid, can in this case also be considered incompressible.

Important Points

- The mass of a fluid is often characterized by its *density* $\rho = m/V$, and its weight is characterized by its *specific weight* $\gamma = W/V$, where $\gamma = \rho g$.
- The *specific gravity* is a ratio of the density or specific weight of a liquid to that of water, defined by $S = \rho/\rho_w = \gamma/\gamma_w$. Here $\rho_w = 1000 \text{ kg/m}^3$ and $\gamma_w = 9.81 \text{ kN/m}^3$.
- For many engineering applications, we can consider a gas to be *ideal*, and can therefore relate its *absolute pressure* to its *absolute temperature* and density using the ideal gas law, $p = \rho RT$.
- The *bulk modulus* of a fluid is a measure of its resistance to compression. Since this property is very high for liquids, we can generally consider liquids as incompressible fluids. Provided a gas has a low velocity of flow—less than 30% of the speed of sound—and has a constant temperature, then the pressure variation within the gas will be low, and we can, under these circumstances, also consider it to be incompressible.

EXAMPLE 1.2

Air contained in the tank, Fig. 1–9, is under an absolute pressure of 60 kPa and has a temperature of 60°C. Determine the mass of the air in the tank.

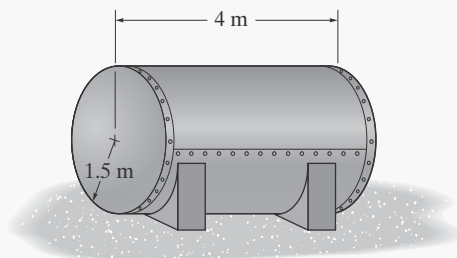


Fig. 1–9

SOLUTION

We will first find the density of the air in the tank using the ideal gas law, Eq. 1–7, $p = \rho RT$. Then, knowing the volume of the tank, we can determine the mass of the air. The *absolute temperature* of the air is

$$T_K = T_C + 273 \text{ K} = 60^\circ\text{C} + 273 \text{ K} = 333 \text{ K}$$

From Appendix A, the gas constant for air is $R = 286.9 \text{ J}/(\text{kg} \cdot \text{K})$. Then,

$$\begin{aligned} p &= \rho RT \\ 60(10^3) \text{ N/m}^2 &= \rho(286.9 \text{ J/kg} \cdot \text{K})(333 \text{ K}) \\ \rho &= 0.6280 \text{ kg/m}^3 \end{aligned}$$

The mass of air within the tank is therefore

$$\begin{aligned} \rho &= \frac{m}{V} \\ 0.6280 \text{ kg/m}^3 &= \frac{m}{[\pi(1.5 \text{ m})^2(4 \text{ m})]} \\ m &= 17.8 \text{ kg} \end{aligned} \quad \text{Ans.}$$

Many people are often surprised by how large the mass of a gas contained within a volume can be. For example, if we repeat the calculations for the mass of air in a typical classroom that measures 4 m by 6 m by 3 m, at a standard room temperature of 20°C and pressure of 101.3 kPa, the result is 86.8 kg. The weight of this air is 851 N. It is no wonder that the flow of air can cause the lift of an airplane and structural damage to buildings.

EXAMPLE 1.3

An amount of glycerin has a volume of 1 m^3 when the pressure is 120 kPa. If the pressure is increased to 400 kPa, determine the change in volume of this cubic meter. The bulk modulus for glycerin is $E_v = 4.52 \text{ GPa}$.

SOLUTION

We must use the definition of the bulk modulus for the calculation. First, the pressure increase applied to the cubic meter of glycerin is

$$\Delta p = 400 \text{ kPa} - 120 \text{ kPa} = 280 \text{ kPa}$$

Thus, the change in volume is

$$E_v = -\frac{\Delta p}{\Delta V/V}$$

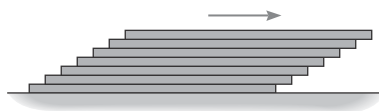
$$4.52(10^9) \text{ N/m}^2 = -\frac{280(10^3) \text{ N/m}^2}{\Delta V/1 \text{ m}^3}$$

$$\Delta V = -61.9(10^{-6}) \text{ m}^3 \quad \text{Ans.}$$

This is indeed a very small change. Since ΔV is directly proportional to the change in pressure, doubling the pressure change will then double the change in volume. Although E_v for water is about half that of glycerin, even for water the volume change will still remain very small!

1.7 Viscosity

Viscosity is a property of a fluid that measures the *resistance to movement* of a very thin layer of fluid over an adjacent one. This resistance occurs only when a tangential or shear force is applied to the fluid, Fig. 1–10a. The resulting deformation occurs at different rates for different types of fluids. For example, water or gasoline will shear or flow faster (low viscosity) than tar or syrup (high viscosity).



(a)

Fig. 1–10

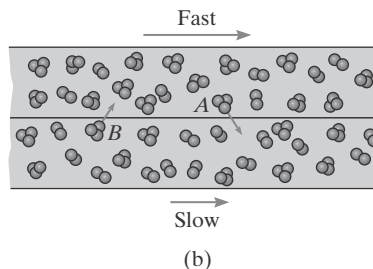
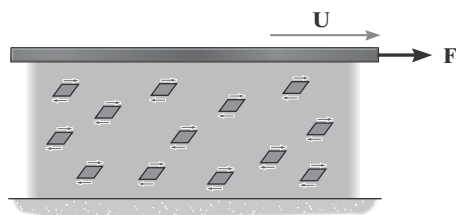


Fig. 1-10 (cont.)

Physical Cause of Viscosity. The resistance that gives rise to viscosity in a fluid can be understood by considering the two layers of fluid in Fig. 1-10*b* sliding past one another. Since the molecules composing the fluid are always in continuous motion, then when molecule *A* in the *faster* top layer happens to travel down to the *slower* bottom layer, it will have a component of motion to the right. Collisions that occur with any slower-moving molecule of the bottom layer will cause it to be *pushed along* due to the momentum exchange with *A*. The reverse effect occurs when molecule *B* in the bottom layer migrates upward. Here this slower-moving molecule will *retard* a faster-moving molecule through their momentum exchange. On a grand scale, both of these effects cause resistance or viscosity.

Newton's Law of Viscosity. To show on a small scale how fluids behave when subjected to a shear force, let us now consider a thin layer of fluid that is confined between a fixed surface and a very wide horizontal plate, Fig. 1-11*a*. When a very *small* horizontal force \mathbf{F} is applied to the plate, it will cause elements of the fluid to distort as shown. After a brief acceleration, the viscous resistance of the fluid will bring the plate into equilibrium, such that the plate will begin to move with a *constant velocity* \mathbf{U} . During this motion, the molecular adhesive force between the fluid particles in contact with *both* the fixed surface and the plate creates a “*no-slip condition*,” such that the fluid particles at the *fixed surface* remain *at rest*, while those on the plate's bottom surface move with the same velocity as the plate.* In between these two surfaces, very thin layers of fluid are dragged along, so that the velocity profile u across the thickness of the fluid will be parallel to the plate, and can vary, as shown in Fig. 1-11*b*.



Distortion of fluid elements due to shear

(a)

Fig. 1-11

*Recent findings have confirmed that this “no-slip condition” is not always true. A fast-moving fluid flowing over an *extremely smooth surface* develops no adhesion. Also, surface adhesion can be reduced by adding *soap-like molecules* to the fluid, which coats the surface, thereby making it extremely smooth. For *most* engineering applications, however, the layer of fluid molecules adjacent to a solid boundary *will adhere to the surface*, and so these special cases with slipping at the boundary will *not* be considered in this text. See Ref. [11].

Shear Stress. The motion just described is a consequence of the shearing effect within the fluid caused by the plate. This effect subjects each element of fluid to a **shear stress** τ (tau), Fig. 1-11c, that is defined as a tangential force ΔF that acts on an area ΔA of the element. It can be expressed as

$$\tau = \lim_{\Delta A \rightarrow 0} \frac{\Delta F}{\Delta A} = \frac{dF}{dA} \quad (1-9)$$

Shear Strain. Since a fluid will flow, this shear stress will cause each element to deform into the shape of a parallelogram, Fig. 1-11c, and during the short time Δt , the resulting deformation is defined by its **shear strain**, specified by the small angle $\Delta\alpha$ (alpha), where

$$\Delta\alpha \approx \tan \Delta\alpha = \frac{\delta x}{\Delta y}$$

A solid would hold this angle under load, but a fluid element will *continue to deform*, and so in fluid mechanics, the *time rate of change in this shear strain (angle)* becomes important. Since the top of the element moves at a rate of Δu relative to its bottom, Fig. 1-11b, then $\delta x = \Delta u \Delta t$. Substituting this into the above equation, the time rate of change of the shear strain becomes

$$\frac{\Delta\alpha}{\Delta t} = \frac{\Delta u}{\Delta y}$$

And in the limit, as $\Delta t \rightarrow 0$,

$$\frac{d\alpha}{dt} = \frac{du}{dy}$$

The term on the right is called the **velocity gradient** because it is an expression of the change in velocity u with respect to y .

In the late 17th century, Isaac Newton proposed that the shear stress in the fluid is directly proportional to this shear strain rate or velocity gradient. This is often referred to as **Newton's law of viscosity**, and it can be written as

$$\tau = \mu \frac{du}{dy} \quad (1-10)$$

The constant of proportionality μ (mu) is a *physical property of the fluid* that measures the *resistance to fluid movement*. Although it is sometimes called the *absolute or dynamic viscosity*, we will refer to it simply as the **viscosity**. From the equation, μ has units of $\text{N} \cdot \text{s}/\text{m}^2$.

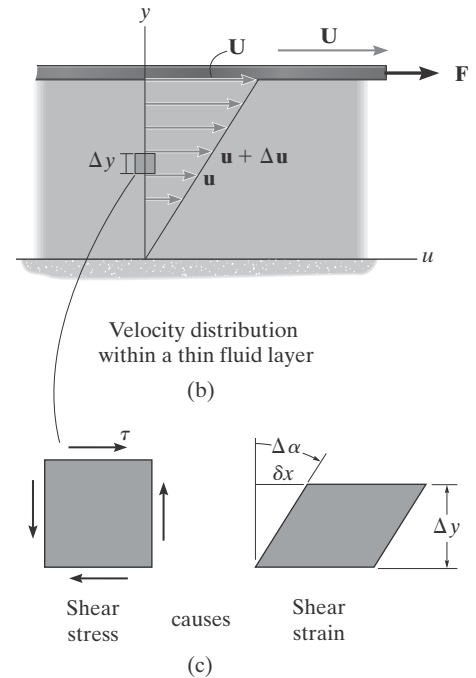
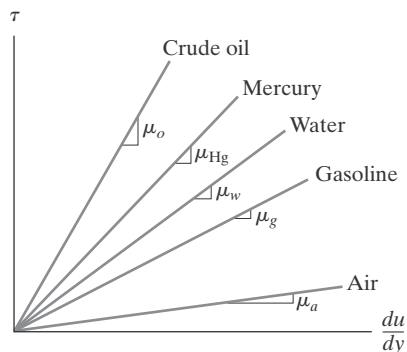


Fig. 1-11 (cont.)



The higher the viscosity, the more difficult it is for a fluid to flow.

Fig. 1-12

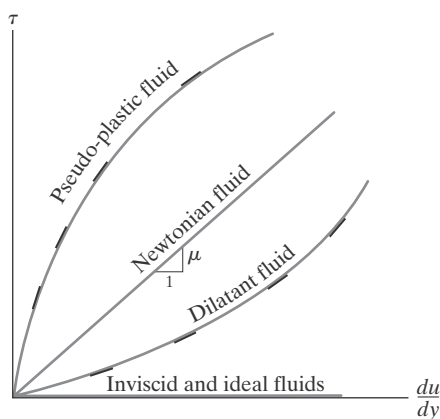
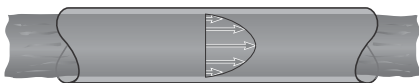
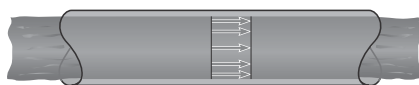


Fig. 1-13



Velocity profile for a real fluid
(a)



Velocity profile for an inviscid or ideal fluid
(b)

Fig. 1-14

Newtonian Fluids. Experiments have shown that many common fluids obey Newton's law of viscosity, and any fluid that does so is referred to as a *Newtonian fluid*. A plot showing how the shear stress and shear-strain rate (velocity gradient) behave for some common Newtonian fluids is shown in Fig. 1-12. Notice how the slope (viscosity) increases, from air, which has a very low viscosity, to water, and then to crude oil, which has a much higher viscosity. In other words, *the higher the viscosity, the more resistant the fluid is to flow*.

Non-Newtonian Fluids. Fluids whose very thin layers exhibit a *nonlinear* behavior between the applied shear stress and the shear-strain rate are classified as *non-Newtonian fluids*. There are basically two types, and they behave as shown in Fig. 1-13. For each of these fluids, the *slope of the curve* for any specific shear-strain rate defines the *apparent viscosity* for that fluid. Those fluids that have an increase in apparent viscosity (slope) with an increase in shear stress are referred to as shear-thickening or *dilatant fluids*. Examples include water with high concentrations of sugar, and quicksand. Many more fluids, however, exhibit the opposite behavior and are called shear-thinning or *pseudo-plastic fluids*. Examples include blood, gelatin, and milk. As noted, these substances flow slowly at low applications of shear stress (large slope), but rapidly under a higher shear stress (smaller slope).

Finally, there exist other classes of substances that have *both* solid and fluid properties. For example, paste and wet cement hold their shape (solid) for small shear stress, but can easily flow (fluid) under larger shear loadings. These substances, as well as other unusual solid–fluid substances, are studied in the field of *rheology*, not in fluid mechanics. See Ref. [8].

Inviscid and Ideal Fluids. Many applications in engineering involve fluids that have *very low viscosities*, such as water and air, [$1.00(10^{-3}) \text{ N} \cdot \text{s}/\text{m}^2$ and $18.1(10^{-6}) \text{ N} \cdot \text{s}/\text{m}^2$, at 20°C] and so we can sometimes *approximate* them as inviscid fluids. By definition, an *inviscid fluid* has *zero viscosity*, $\mu = 0$, and as a result it offers *no resistance to shear stress*, Fig. 1-13. In other words, it is *frictionless*. Hence, if the fluid in Fig. 1-11 is inviscid, then when the force \mathbf{F} is applied to the plate, it will cause the plate to continue to *accelerate*, since no shear stress can be developed within an inviscid fluid to offer a restraining frictional resistance to the bottom of the plate. If in addition to being inviscid, the fluid is also assumed to be incompressible, then it is called an *ideal fluid*. By comparison, if any *real fluid* flows slowly through a pipe, it will have a velocity profile that looks something like that in Fig. 1-14a, whereas an inviscid or ideal fluid will have a uniform velocity profile, Fig. 1-14b.

Pressure and Temperature Effects. Through experiment it has been found that the viscosity of a fluid is actually *increased* with pressure, although this effect is quite small and so it is generally neglected for most engineering applications. Temperature, however, affects the

viscosity of fluids to a much greater extent. In the case of a *liquid*, an *increase in temperature* will *decrease its viscosity*, as shown in Fig. 1–15 for water and mercury; Ref. [9]. This occurs because a temperature increase will cause the molecules of the liquid to have more vibration or mobility, thus breaking their molecular bonds and allowing the layers of the liquid to “loosen up” and slip more easily. If the fluid is a gas, an *increase in temperature* has the opposite effect, that is, the *viscosity will increase* as noted for air and carbon dioxide in Fig. 1–15; Ref. [10]. Since gases are composed of molecules that are much farther apart than for a liquid, their intermolecular attraction to one another is *smaller*. When the temperature increases the molecular motion of the gas will increase, and this will increase the momentum *exchange* between successive layers. It is this additional resistance, developed by molecular collisions, that causes the viscosity to increase.

Attempts have been made to use empirical equations to fit the experimental curves of viscosity versus temperature for various liquids and gases, such as those shown in Fig. 1–15. For liquids, the curves can be represented using *Andrade’s equation*.

$$\mu = Be^{C/T} \text{ (liquid)}$$

And for gases, the *Sutherland equation* works well.

$$\mu = \frac{BT^{3/2}}{(T + C)} \text{ (gas)}$$

In each of these cases T is the *absolute temperature*, and the constants B and C can be determined if specific values of μ are known for two different temperatures.*

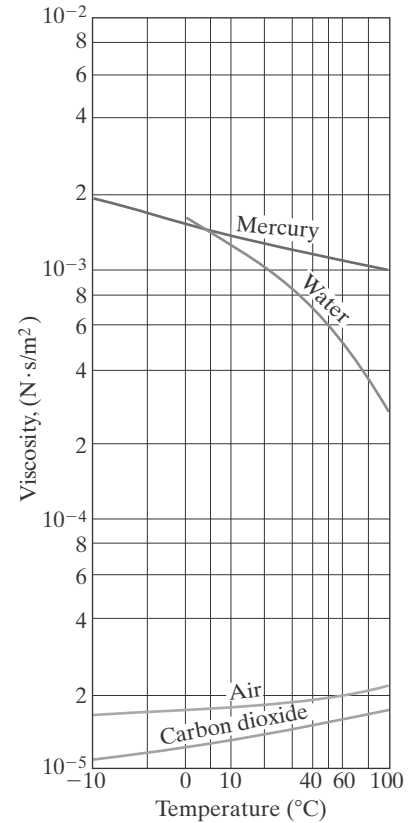
Kinematic Viscosity. Another way to express the viscosity of a fluid is to represent it by its *kinematic viscosity*, ν (nu), which is the ratio of its dynamic viscosity to its density.

$$\nu = \frac{\mu}{\rho} \quad (1-11)$$

The units are m^2/s .** The word “kinematic” is used to describe this property because force is not involved in the dimensions. Typical values of the dynamic and kinematic viscosities are given in Appendix A for some common liquids and gases, and more extensive listings are also given for water and air.

*See Probs. 1–30 and 1–33.

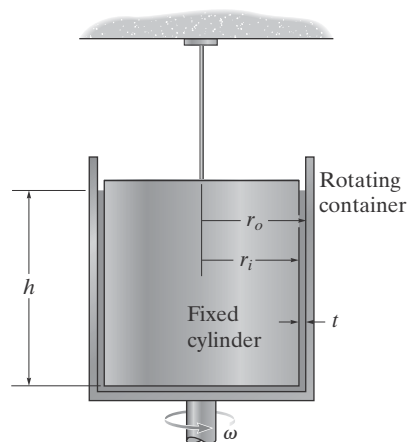
**In the standard metric system (not SI), grams and centimeters (100 cm = 1 m) are used. In this case the dynamic viscosity μ is expressed using a unit called a *poise*, where $\text{poise} = 1 \text{ g}/(\text{cm} \cdot \text{s})$, and the kinematic viscosity ν is measured in *stokes*, where 1 stoke = $1 \text{ cm}^2/\text{s}$.



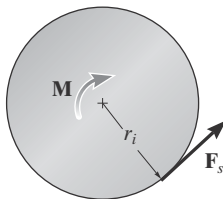
Viscosity vs. temperature

Fig. 1–15

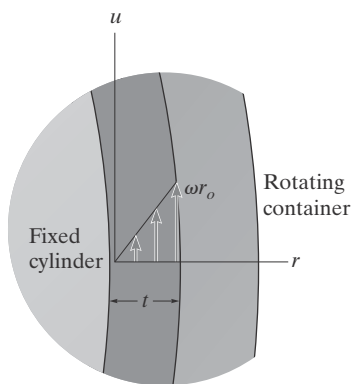
1.8 Viscosity Measurement



(a)



(b)



(c)

Fig. 1-16

The viscosity of a Newtonian liquid can be measured in several ways. One common method is to use a **rotational viscometer**, sometimes called a *Brookfield viscometer*. This device, shown in the photo on the next page, consists of a solid cylinder that is suspended within a cylindrical container as shown in Fig. 1-16a. The liquid to be tested fills the small space between these two cylinders, and as the container is forced to rotate with a very slow constant angular velocity ω , it causes the contained cylinder to twist the suspension wire a small amount before it attains equilibrium. By measuring the angle of twist of the wire, the torque M in the wire can be calculated using the theory of mechanics of materials. This torque resists the moment caused by the shear stress exerted by the liquid on the surface of the suspended cylinder. Once this torque is known, we can then find the viscosity of the fluid using Newton's law of viscosity.

To demonstrate how this is done, consider only the effect of shear stress developed on the vertical surface of the cylinder.* We require M , the torque in the wire, to balance the moment of the resultant shear force the liquid exerts on the cylinder's surface about the axis of the cylinder, Fig. 1-16b. This gives $F_s = M/r_i$. Since the area of the surface is $(2\pi r_i)h$, the shear stress acting on the surface is

$$\tau = \frac{F_s}{A} = \frac{M/r_i}{2\pi r_i h} = \frac{M}{2\pi r_i^2 h}$$

The angular rotation of the container causes the liquid in contact with its wall to have a speed of $U = \omega r_o$, Fig. 1-16c. Since the suspended cylinder is held stationary by the wire once the wire is fully twisted, and because the gap t is *very small*, the velocity gradient across the thickness t of the liquid can be assumed to be constant. If this is the case, it can then be expressed as

$$\frac{du}{dr} = \frac{\omega r_o}{t}$$

Using Newton's law of viscosity,

$$\tau = \mu \frac{du}{dr}; \quad \frac{M}{2\pi r_i^2 h} = \mu \frac{\omega r_o}{t}$$

Solving for μ in terms of the measured properties, the viscosity is then

$$\mu = \frac{Mt}{2\pi\omega r_i^2 r_o h}$$

*An extended analysis includes the frictional resistance of the liquid on the cylinder's bottom surface. See Probs. 1-50 and 1-51.

The viscosity of a liquid can also be obtained by using other methods. For example, W. Ostwald invented the *Ostwald viscometer* shown in the photo at the bottom of the page. Here the viscosity is determined by measuring the time for a liquid to flow through the short, small-diameter tube, and then correlating this time with the time for another liquid of known viscosity to flow through this same tube. The unknown viscosity is then determined by direct proportion. Another approach is to measure the speed of a small sphere as it falls through the liquid that is to be tested. It will be shown in Sec. 11.8 that this speed can be related to the viscosity of the liquid. Such an approach works well for transparent liquids, such as honey, which have a very high viscosity. In addition, many other devices have been developed to measure viscosity, and the details on how they work can be found in books related to this subject. For example, see Ref. [14].

Important Points

- A *Newtonian fluid*, such as water, oil, or air, develops shear stress within successive thin layers of the fluid that is directly proportional to the velocity gradient that occurs between the fluid layers, $\tau = \mu (du/dy)$.
- The shear resistance of a Newtonian fluid is measured by the proportionality constant μ , called the viscosity. The higher the viscosity, the greater the resistance to flow caused by shear.
- A non-Newtonian fluid has an apparent viscosity. If the apparent viscosity increases with an increase in shear stress, then the fluid is a dilatant fluid. If the apparent viscosity decreases with an increase in shear stress, then it is a pseudo-plastic fluid.
- An inviscid fluid has no viscosity, and an ideal fluid is both inviscid and incompressible; that is, $\mu = 0$ and $\rho = \text{constant}$.
- The viscosity varies only slightly with pressure; however, for increasing temperature, μ will decrease for liquids, but it will increase for gases.
- The kinematic viscosity ν is the ratio of the two fluid properties ρ and μ , where $\nu = \mu/\rho$.
- It is possible to obtain the viscosity of a liquid in an indirect manner by using a rotational viscometer, an Ostwald viscometer, or by several other methods.



Brookfield viscometer



Ostwald viscometer

EXAMPLE 1.4

The plate in Fig. 1–17 rests on top of the thin film of water, which is at a temperature of 25°C. When a small force \mathbf{F} is applied to the plate, the velocity profile across the thickness of the fluid can be described as $u = (40y - 800y^2)$ m/s, where y is in meters. Determine the shear stress acting on the fixed surface and on the bottom of the plate.

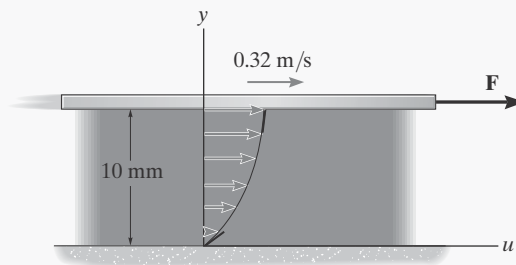


Fig. 1–17

SOLUTION

Fluid Description. Water is a Newtonian fluid, and so Newton's law of viscosity applies. The viscosity of water at 25°C is found from Appendix A to be $\mu = 0.897(10^{-3}) \text{ N} \cdot \text{s}/\text{m}^2$.

Analysis. Before applying Newton's law of viscosity, we must first obtain the velocity gradient.

$$\frac{du}{dy} = \frac{d}{dy} (40y - 800y^2) \text{ m/s} = (40 - 1600y) \text{ s}^{-1}$$

Therefore, at the fixed surface, $y = 0$,

$$\begin{aligned} \tau &= \mu \left. \frac{du}{dy} \right|_{y=0} = (0.897(10^{-3}) \text{ N} \cdot \text{s}/\text{m}^2)(40 - 0) \text{ s}^{-1} \\ \tau &= 35.88(10^{-3}) \text{ N}/\text{m}^2 = 35.9 \text{ mPa} \end{aligned} \quad \text{Ans.}$$

And, at the bottom of the moving plate, $y = 0.01$ m,

$$\begin{aligned} \tau &= \mu \left. \frac{du}{dy} \right|_{y=0.01 \text{ m}} = [0.897(10^{-3}) \text{ N} \cdot \text{s}/\text{m}^2](40 - 1600(0.01)) \text{ s}^{-1} \\ \tau &= 21.5 \text{ mPa} \end{aligned} \quad \text{Ans.}$$

By comparison, the *larger shear stress* develops on the fixed surface rather than on the bottom of the plate since the *velocity gradient* or slope du/dy is *large* at the fixed surface. Both of these slopes are indicated by the short dark lines in Fig. 1–17. Also, notice that the equation for the velocity profile must satisfy the boundary condition of no slipping, i.e., at the fixed surface $y = 0$, $u = 0$, and with the movement of the plate at $y = 10$ mm, $u = U = 0.32$ m/s.

EXAMPLE 1.5

The 100-kg plate in Fig. 1–18a is resting on a very thin film of SAE 10W-30 oil, which has a viscosity of $\mu = 0.0652 \text{ N}\cdot\text{s}/\text{m}^2$. Determine the force \mathbf{P} that must be applied to the center of the plate to slide it over the oil with a constant velocity of 0.2 m/s. Assume the oil thickness is 0.1 mm, and the velocity profile across this thickness is linear. The bottom of the plate has a contact area of 0.75 m^2 with the oil.

SOLUTION

Fluid Description. The oil is a Newtonian fluid, and so Newton's law of viscosity can be applied.

Analysis. First we draw the free-body diagram of the plate in order to relate the shear force \mathbf{F} caused by the oil on the bottom of the plate to the applied force \mathbf{P} , Fig. 1–18b. Because the plate moves with constant velocity, the force equation of equilibrium in the horizontal direction applies.

$$\begin{aligned} \rightarrow \Sigma F_x &= 0; & F - P \cos 30^\circ &= 0 \\ & & F &= 0.8660P \end{aligned}$$

The effect of this force *on the oil* is in the opposite direction, and so the *shear stress* on the top of the oil acts to the left. It is

$$\tau = \frac{F}{A} = \frac{0.8660P}{0.75 \text{ m}^2} = (1.155P) \text{ m}^{-2}$$

Since the velocity profile is assumed to be linear, Fig. 1–18c, the velocity gradient is constant, $du/dy = U/t$, and so

$$\tau = \mu \frac{du}{dy} = \mu \frac{U}{t}$$

$$(1.155P) \text{ m}^{-2} = (0.0652 \text{ N}\cdot\text{s}/\text{m}^2) \left[\frac{0.2 \text{ m/s}}{0.1(10^{-3}) \text{ m}} \right]$$

$$P = 113 \text{ N}$$

Ans.

Notice that the constant velocity gradient will produce a constant shear-stress distribution across the thickness of the oil, which is $\tau = \mu (U/t) = 130 \text{ Pa}$, Fig. 1–18c.

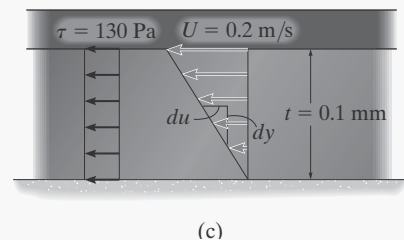
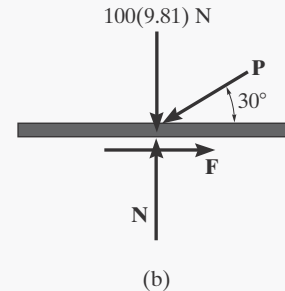
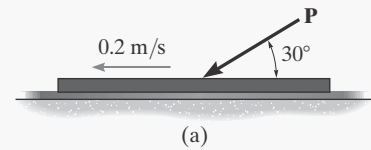
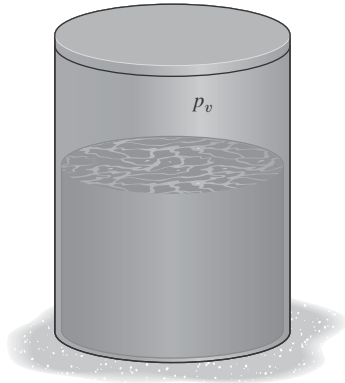


Fig. 1–18

1.9 Vapor Pressure



The vapor pressure p_v will form within the top space of the closed tank that was originally a vacuum

Fig. 1–19

Consider a liquid contained in a closed tank, as in Fig. 1–19. Because the temperature of the liquid will cause continuous thermal agitation of the liquid's molecules, some of these molecules near the surface will acquire enough kinetic energy to break their molecular bonds with adjacent molecules, and will move upward or evaporate into the empty space of the tank. When a state of equilibrium is reached, the number of molecules that evaporate from the liquid will equal the number of molecules that condense back to it. The empty space is then said to be **saturated**. By bouncing off the walls of the tank and the liquid surface, the evaporated molecules create a pressure within the tank. This pressure is called the **vapor pressure**, p_v . Any increase in liquid temperature will increase the rate of evaporation, and also the kinetic energy of the liquid's molecules, so higher temperatures will cause higher vapor pressures.

The liquid will begin to boil when the absolute pressure at its surface is at or lower than its *vapor pressure*. For example, if water at sea level is brought to a temperature of 100°C, then at this temperature its vapor pressure will equal the atmospheric pressure, which is 101.3 kPa, and so the water will boil. In a similar manner, if the atmospheric pressure at the water surface is *reduced*, such as at the top of a mountain, then boiling occurs at this lower pressure, when the temperature is less than 100°C. Specific values of the vapor pressure for water at various temperatures are given in Appendix A. Notice that as the temperature increases, so does the vapor pressure due to the increase in the thermal agitation of its molecules.

Cavitation. When engineers design pumps, turbines, or piping systems, it is important that they do not allow the liquid at any point within the flow to be subjected to a pressure *equal to or less than* its vapor pressure. If this occurs as stated above, rapid evaporation or boiling will occur within the liquid. The resulting bubbles will migrate to regions of higher pressure, and then suddenly collapse, creating a phenomenon known as **cavitation**. The repeated pounding caused by this effect against the surface of a propeller blade or pump casing can eventually wear down its surface, and so it is important to avoid its occurrence. Later, in Chapter 14, we will study the significance of cavitation in greater detail.

1.10 Surface Tension and Capillarity

A liquid maintains its form because its molecules are attracted to one another by **cohesion**. It is this force that enables liquids to resist tensile stress and thereby creates *surface tension* in the liquid. On the other hand, if liquid molecules are attracted to those of a different substance, the force of attraction is known as **adhesion**, and this force, along with that of cohesion, gives rise to *capillarity*.

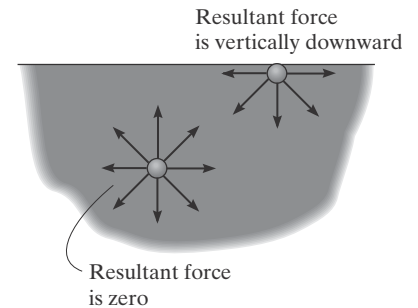
Surface Tension. The phenomenon of surface tension can be explained by visualizing the cohesive forces acting on two molecules (or particles) in a liquid, shown in Fig. 1–20a. The molecule located deep within the liquid has the same cohesive forces acting on it by all the surrounding molecules. Consequently, there is no resultant force acting on it. However, the molecule located on the surface of the liquid has cohesive forces that come only from molecules that are next to it on the surface and from those below it. This will produce a net resultant downward force, and the effect of all such forces will produce a *contraction* of the surface. In other words, the resultant cohesive force attempts to pull downward on the surface.

To separate the molecules at the surface requires a tensile force. We call this tensile force per unit length in any direction along the surface the **surface tension**, σ (sigma), Fig. 1–20b. It has units of N/m, and for any liquid, its value depends primarily upon the temperature. The higher the temperature, the more thermal agitation occurs, and so the surface tension becomes smaller. For example, water at 10°C has $\sigma = 74.2$ mN/m, whereas at the higher temperature of 50°C, $\sigma = 67.9$ mN/m. Values of σ , such as these, are sensitive to impurities, so care should be taken when using published values.

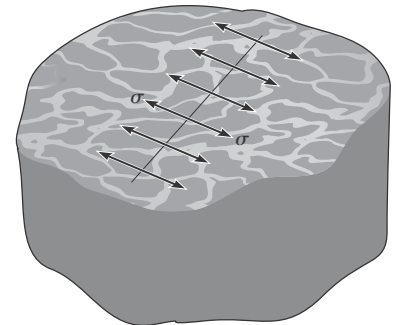
Because cohesion resists any increase in the surface area of a liquid, it actually tries to *minimize* the size of the surface. Separating the molecules and thus breaking the surface tension requires work, and the energy produced by this work is called **free-surface energy**. For example, suppose a small element of the surface is subjected to the surface tension force $F = \sigma \Delta y$ along one of its sides, as shown in Fig. 1–20c. If the surface stretches δx , then the increase in the area is $\Delta y \delta x$. The force \mathbf{F} does work of $F \delta x$, and so the work done per area increase is therefore

$$\frac{F \delta x}{\Delta y \delta x} = \frac{\sigma \Delta y \delta x}{\Delta y \delta x} = \sigma$$

In other words, the surface tension can also be thought of as the amount of free-surface energy required to increase a unit surface area of a liquid.

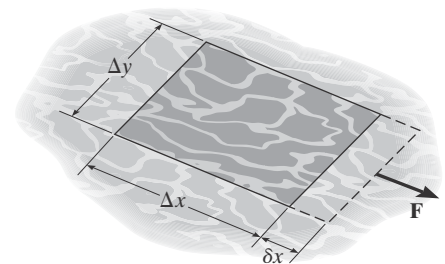


(a)



(b)

Surface tension is the force per unit length needed to separate the molecules on the surface



(c)

Fig. 1–20



Like rain drops, the water ejected from this fountain forms spherical droplets due to the cohesive force of surface tension.

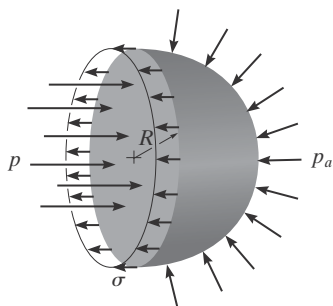


Fig. 1-21

Liquid Drops. Cohesion is responsible for the formation of liquid droplets that naturally form when a liquid is sprayed into the atmosphere. The cohesion minimizes the shape of any water droplet, and so it forms a sphere. We can determine the pressure that cohesion causes within a droplet provided we know the surface tension σ for the liquid. To do this, consider the free-body diagram of half the drop, Fig. 1-21. If we neglect gravity and the effects of atmospheric drag as the drop falls, then the only forces acting are those due to atmospheric pressure, p_a , on its *outside* surface; surface tension, σ , around the *surface* of the drop where it is sectioned; and the internal pressure, p , on the sectioned area. As will be explained in the next chapter, the resultant horizontal forces due to p_a and p are determined by multiplying each pressure by the *projected area* of the drop, that is, πR^2 , and the resultant force of the surface tension is determined by multiplying σ by the circumferential distance around the drop, $2\pi R$. For horizontal equilibrium, we therefore have

$$\pm \Sigma F_x = 0; \quad p(\pi R^2) - p_a(\pi R^2) - \sigma(2\pi R) = 0$$

$$p = \frac{2\sigma}{R} + p_a$$

Here the internal pressure is composed of two parts, one due to surface tension and the other due to atmospheric pressure. For example, mercury at a temperature of 20°C has a surface tension of $\sigma = 486 \text{ mN/m}$. If the mercury forms into a 2-mm-diameter drop, its surface tension will create an internal pressure of $p_{st} = 2(0.486 \text{ N/m}) / (0.001 \text{ m}) = 972 \text{ Pa}$ within the drop, in addition to the pressure caused by the atmosphere.

Capillarity. The capillarity of a liquid depends upon the comparison between the forces of adhesion and cohesion. If the force of a liquid's adhesion to the molecules of the surface of its container is *greater* than the force of cohesion between the liquid's molecules, then the liquid is referred to as a **wetting liquid**. In this case, the **meniscus** or surface of the liquid, such as water in a narrow glass container, will be concave, Fig. 1-22a. If the adhesive force is *less* than the cohesive force, as in the case of mercury, then the liquid is called a **nonwetting liquid**. The meniscus forms a convex surface, Fig. 1-22b.



Mercury is a nonwetting liquid as noted by the way its edge curls inward.

Wetting liquids will rise up along a narrow tube, Fig. 1–23a, and we can determine this height h by considering a free-body diagram of the portion of the liquid suspended in the tube, Fig. 1–23b. Here the free surface or meniscus makes a contact angle θ between the sides of the tube and the liquid surface. This angle defines the direction of the force of adhesion, which is the effect of the surface tension σ of the liquid as it holds the liquid surface up against the wall of the tube. The resultant of this force, which acts around the inner circumference of the tube, is therefore $\sigma(2\pi r) \cos \theta$. The other force is the weight of the suspended liquid, $W = \rho g \mathcal{V}$, where $\mathcal{V} = \pi r^2 h$. For vertical equilibrium, we require

$$+\uparrow \Sigma F_y = 0; \quad \sigma(2\pi r) \cos \theta - \rho g(\pi r^2 h) = 0$$

$$h = \frac{2\sigma \cos \theta}{\rho g r}$$

Experiments have shown that the contact angle between water and glass is $\theta \approx 0^\circ$, and so for water the meniscus surface, shown in Fig. 1–23a, actually becomes somewhat hemispherical. By carefully measuring h , the above equation can then be used with $\theta = 0^\circ$ to determine the surface tension σ for water at various temperatures.

In the next chapter we will show how to determine pressure by measuring the height of a liquid in a glass tube. When it is used for this purpose, however, errors due to the additional height caused by capillarity within the tube can occur. To minimize this effect, notice that h in the above result is inversely proportional to the density of the liquid and the radius of the tube. The smaller they are, the higher h becomes. For example, for a 3-mm-diameter tube containing water at 20°C , where $\sigma = 72.7 \text{ mN/m}$ and $\rho = 998.3 \text{ kg/m}^3$, we have

$$h = \frac{2(0.0727 \text{ N/m}) \cos 0^\circ}{(998.3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.0015 \text{ m})} = 9.90 \text{ mm}$$

This is somewhat significant, and so for experimental work it is generally preferable to use tubes having a diameter of 10 mm or greater, since at 10 mm, $h \approx 3 \text{ mm}$, and the effect of capillarity is minimized.

Throughout our study of fluid mechanics, we will find that, for the most part, the forces of cohesion and adhesion will be small compared to the effects of gravity, pressure, and viscosity. Surface tension generally becomes important, however, when we want to study phenomena related to bubble formation and growth, examine movement of liquids through porous media such as soil, or consider the effects of liquid films on surfaces.

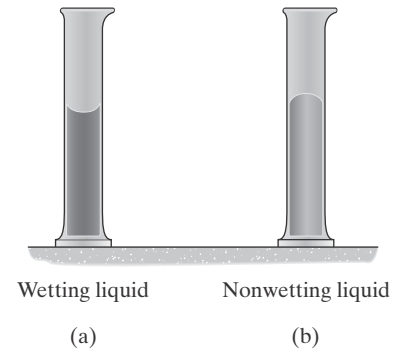


Fig. 1–22

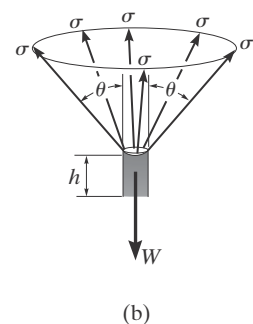
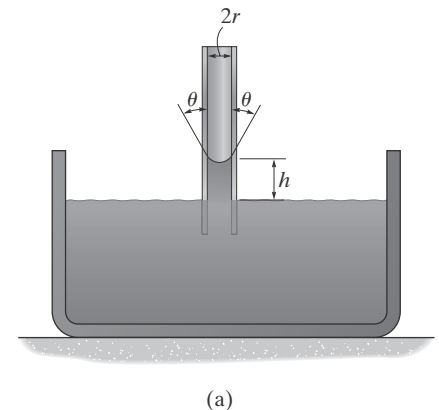


Fig. 1–23

Important Points

- A liquid will begin to boil at a specific temperature when the pressure within it, or at its surface, is equal to its vapor pressure at that temperature.
- Consideration must be given to the possibility of *cavitation* when designing mechanical or structural elements operating within a fluid environment. This phenomenon is caused when the pressure within the fluid is equal to or less than the vapor pressure, causing boiling, migration of the resulting bubbles to a region of higher pressure, and then their sudden collapse.
- *Surface tension*, σ , in a liquid is caused by molecular cohesion. It is measured as a force per unit length acting on the liquid's surface. It becomes smaller as the temperature rises.
- Capillarity of a *wetting liquid*, such as water in a narrow glass tube, creates a concave surface since the force of adhesion to the walls of the tube will be greater than the force caused by the cohesion of the liquid. For a *nonwetting liquid*, such as mercury, the surface is convex since the force of cohesion will be greater than that of adhesion.

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